HUMAN CAPITAL AS THE SOURCE OF ECONOMIC GROWTH

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Abstract

This paper revisits the theoretical framework of endogenous economic growth by considering models where human capital accumulation is at the center of the growth process. Our work is in line with Lucas (2015), who calls for not giving "too large a role to exogenous technological change" (p. 86) while advocating that "the contribution of human capital accumulation to economic growth deserves a production function of its own" (p. 87). The main finding of our research provides the long-term behavior of economies, where our main results locate and extend these discussions to infinite-horizon models in several ways.

Keywords: Dynamic programming; Endogenous growth; Human capital.

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1. INTRODUCTION

By consensus, economic growth comes from the accumulation of physical capital and labor inputs and total factor productivity (TFP). A famous study, the seminal work of Solow (1956, 1957), shows that the contribution of TFP to growth in the United States for the first 50 years of the 20^{th} century was more than 87%.

The approach taken in this paper is that physical capital is represented by the number of machines, while labor force refers to the number of workers and their working time. Therefore, this paper considers TFP in the production process, which gives us adequate capital and labor. TFP is the efficiency of technologies and the skill level of workers. A good understanding of the evolution of these factors and the conditions allowing their sustained growth is of great importance for the long-term dynamics of the economy (Galor & Moav, 2004; Manuelli & Seshadri, 2014; McGrattan & Prescott, 2009; Schoellman, 2012).

Additionally, our research aligns with Lucas (1988), who points out the importance of human capital accumulation in economic growth. Lucas shows that economic growth rates are more significant when effective positive externalities are generated by human capital accumulation. Recently, Lucas (2015), in another seminal article, advocates a central role for human capital accumulation rather than external factors such as technology. Lucas argues there may be "a misinterpretation of the evidence, especially of census data on schooling and age-earnings profiles" (p. 85). In conclusion, Lucas (2015) states that "the contribution of human capital accumulation to economic growth deserves a production function of its own" (p. 87). Our paper attempts to understand the theoretical framework for the role of human capital in long-term dynamics.

Another aspect that this study approaches is understanding that human capital accumulation represents fundamental features that are very different from physical capital accumulation. While the latter are understood as the addition of investment to the existing stock, an increase in human capital depends on the resources invested, learning time, and the level of human capital in combination with the physical capital stock. These differences represent challenges for an in-depth analysis combining all features. Human capital accumulation may happen through investment, learning time, or both. Furthermore, we can consider the accumulations of human and physical capital separately or together. In this article, we will analyze all these configurations and give the characteristics of long-term dynamics in each case. We will also specify precise conditions for ensuring sustained growth.

Moreover, this research discusses the interactions between physical and human capital in different stages of economic growth. We follow studies by McGrattan and Prescott (2009), Schoellman (2012), and Manuelli and Seshadri (2014), who suggest that production factors have a more critical impact on economic growth than external factors such as technology. Also, our research aligns with Galor and Moav (2004), who show that at some stages of economic development, human capital formation can outperform physical capital accumulation as the primary source of economic growth.

The paper is organized as follows. Sections 2 and 3 provide the long-term behavior of economies, where our main results locate and extend these discussions to infinite-horizon models in several ways. Section 4 concludes. Proofs are given in the appendices.

2. FORMATION OF HUMAN CAPITAL ACCUMULATION DEPENDING ON INVESTMENT

Denote the utility function, which is strictly increasing and concave, by $u: \mathbb{R}_+ \to \mathbb{R}$. Given a consumption stream $(c_0, c_1, \dots,)$, the intertemporal utility of the economic agent is

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1}$$

where β belongs to $(0,1)$.

The production function $f: \mathbb{R}_+ \to \mathbb{R}_+$ depends only on the human capital stock and is assumed to be strictly increasing, concave, and to satisfy the Inada conditions.

The economic agent maximizes her constrained intertemporal utility depending on the production and human capital formation functions. In this section, we need to pay more attention to physical capital to simplify the analysis and fix ideas. Instead, the output is created using only human capital, whose accumulation function depends on individuals' saving decisions and the existing stock of knowledge, according to the terms used by Ramsey (1928) and Lucas (2015). The saving decisions can take the form of economic savings or time and effort devoted to enhancing effective labor.

For each time t, the human capital stock and the investment in its formation are denoted by h_t and s_{t+1}, respectively. The evolution of human capital is described as

$$\frac{h_{t+1}}{h_t} = \phi(s_{t+1}), \tag{2}$$

where ϕ represents the formation of the human capital function, which is assumed to be strictly increasing, strictly concave, and to satisfy the Inada conditions. It is intuitive to suppose that $\phi(0) = 1 - \delta$, in which $0 < \delta < 1$ is the depreciation rate of human capital.

The output per capita $y_t = f(h_t)$ is divided between consumption c_t and investment in human capital formation s_{t+1}. For a given initial accumulation of human capital $h_0 > 0$, the economic agent solves

$$\max\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right], \tag{3}$$

$$s.t.\ \ c_t + s_{t+1} \le f(h_t), \tag{4}$$

$$\frac{h_{t+1}}{h_t} = \phi(s_t), \tag{5}$$

$$c_t, s_t \ge 0\ \ for\ \ all\ \ t. \tag{6}$$

Using the same approach as in Stokey et al. (1989), we transform the initial problem into a well-known one in the dynamic programming literature. Denote by ψ the inverse function of ϕ. Then, for any $h \ge 0$,

$$\psi(\phi(h)) = h. \tag{7}$$

Let

$$\Gamma(h) = \left\{h'\ \ such\ \ that\ \ (1-\delta)h \le h' \le f\big(\phi(h)\big)\right\}. \tag{8}$$

It is easy to verify that the sequence of human capital accumulation $\{h_t\}_{t=0}^{\infty}$ is feasible if and only if $h_{t+1} \in \Gamma(h_t)$ for any $t \ge 0$. For h and h' such that $h' \in \Gamma(h)$, define the indirect utility function V as

$$V(h, h') = u\left(f(h) - \psi\left(\frac{h'}{h}\right)\right). \tag{9}$$

For a given $h_0 > 0$, the initial program can be rewritten as

$$\max\left[\sum_{t=0}^{\infty}\beta^t V(h_t, h_{t+1})\right], \tag{10}$$

$$s.t.\ \ h_{t+1} \in \Gamma(h_t)\ \ for\ \ any\ \ t \ge 0. \tag{11}$$

Although this economy's utility, production, and formation functions satisfy every usual concavity property, the problem is not convex, and we must consider the possibility of multiple solutions.

Ha-Huy and Tran (2020) prove that the indirect utility function V satisfies the supermodularity[1] property, as presented in Amir (1996). The supermodularity property implies that every optimal path is either strictly monotonic or constant. We verify that the

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[1] The (strict) supermodularity is defined as: for every (x, x') and (y, y') that belong to $Graph\,(\Gamma)$, $V(x,y) + V(x',y')(>) \ge V(x',y) + V(x,y')$ is verified whenever $(x',y')(>) \ge (x,y)$. When V is twice differentiable, (strict) supermodularity sums up to positive cross derivatives: $V_{12}(x,y)(>) \ge 0$ for every x, y.

set of initial accumulation stock h_0 allows multiple solutions of measure zero. Hence, there exists a unique optimal path that converges to a finite value or diverges to infinity.

Since $\phi(0) < 1$, for a low value of h_0, we have $h_1 < h_0$. Hence, the optimal path $\{h_t^*\}_{t=0}^{\infty}$ is strictly decreasing, and the economy converges to zero.

The following question remains: is it true that for the initial state sufficiently great, the economy can diverge to infinity or at least be bounded away from zero? The answer to this question is affirmative.

In Ha-Huy and Tran (2020), the authors consider the following inequality

$$V_2(h,h) + \beta V_1(h,h) > 0, \tag{12}$$

where V_1 and V_2 are the partial derivatives corresponding to the first and the second arguments, respectively. The intuitive basis of this inequality is that, at level h, between saving a little more and maintaining the "*status quo*," the decision to save more prevails. Under such conditions, satisfying this inequality for a sufficiently large value of h ensures the economy diverges to infinity.

Some simple calculus gives

$$V_2(h,h) + \beta V_1(h,h) = u'\big(f(h) - \psi(1)\big)\left(\beta f'(h) - \frac{(1-\beta)\psi'(1)}{h}\right). \tag{13}$$

The inequality

$$V_2(h,h) + \beta V_1(h,h) > 0 \tag{14}$$

is equivalent to the following:

$$f'(h) > \frac{(1-\beta)\psi'(1)}{\beta h}. \tag{15}$$

Although this inequality is not satisfied for a small value of human capital stock[2], h, it is generally true for a large value of h, especially if the marginal productivity of f remains bounded away from zero, even for a high level of human capital accumulation.

Proposition 2.1. *If $f(h) = h^{\gamma}$, there exists $\bar{h}$ large enough such that for any $h_0 > \bar{h}$, any optimal path beginning at h_0 is increasing and diverges to infinity.*

Proof. See Appendix A.

[2] Indeed, for h sufficiently small, we have $hf'(h) \leq f(h) < (1-\beta)\psi'(1)$.

3. FORMATION OF HUMAN CAPITAL ACCUMULATION DEPENDING ON LEARNING TIME

This section considers the discrete-time versions of Lucas (1988) and Caballé and Santos (1993). This research is in line with studies by Stokey et al. (1989) and Le Van and Dana (2003). Suppose that time for workers is normalized to 1 and can be divided into two parts: time devoted to work, denoted by τ_t, and time devoted to human capital formation (learning, training, relaxing, leisure, health, etc.). The following equation characterizes the accumulation of human capital over time

$$\frac{h_{t+1}}{h_t} = 1 - \delta + \theta(1 - \tau_t), \tag{16}$$

where $\theta > 0$ is a parameter capturing the efficiency of the training (or recovery) process. At time t, given working time, τ_t, and human capital level, h_t, the output is

$$y_t = f(\tau_t h_t). \tag{17}$$

All output is consumed: $c_t = f(\tau_t h_t)$. Given an initial level $h_0 > 0$, the economic agent solves

$$\max \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right], \tag{18}$$

$$s.t.\ \ c_t = f(\tau_t h_t), \tag{19}$$

$$h_{t+1} = [1 - \delta + \theta(1 - \tau_t)]h_t\,, for\ \ all\ \ \ t. \tag{20}$$

We now transform the problem into a well-known one. First, observe that for any $t \geq 0$, we have

$$\tau_t h_t = \frac{(1 - \delta + \theta)h_t - h_{t+1}}{\theta}. \tag{21}$$

For any $(1 - \delta)h \leq h' \leq (1 - \delta + \theta)h$, let us define the indirect utility function as

$$V(h, h') = u\left(f\left(\frac{(1 - \delta + \theta)h - h'}{\theta}\right)\right). \tag{22}$$

For each $h_0 > 0$, we consider the following program, which is equivalent to the initial one

$$\max \left[\sum_{t=0}^{\infty} \beta^t V(h_t, h_{t+1}) \right], \tag{23}$$

$$s.t.\ \ (1-\delta)h_t \leq h_{t+1} \leq (1-\delta+\theta)h_t \ \ for\ \ all\ \ t. \tag{24}$$

Obviously, in the case where $\theta < \delta$, every feasible path converges to zero, and the problem becomes trivial. In this situation, human capital formation needs to be more efficient to compensate for human capital depreciation.

Now, assume that $\theta \geq \delta$. It is easy to verify that the function V satisfies the supermodularity property. Moreover, since V is concave, the problem satisfies the usual convexity properties in the dynamic programming literature. Hence, for all $h_0 > 0$, the solution is unique and monotonic.

We consider the inequality in Ha-Huy and Tran (2020), which is

$$V_2(h,h) + \beta V_1(h,h) > 0. \tag{25}$$

After some simple calculus, we get

$$V_2(h,h) + \beta V_1(h,h) = u'\left(f\left(\frac{\theta-\delta}{\theta}h\right)\right)f'\left(\frac{\theta-\delta}{\theta}h\right) \times \frac{\beta(1-\delta+\theta)-1}{\theta}. \tag{26}$$

As the functions u and f are strictly increasing and both have strictly positive derivatives, the inequality is equivalent to

$$\beta(1-\delta+\theta) > 1. \tag{27}$$

Since the problem is convex, we obtain the proposition below, following Ha-Huy and Tran (2020).

Proposition 3.1.

i. For all $h_0 > 0$, the optimal path is unique.

ii. If $\beta(1-\delta+\theta) > 1$, then the optimal path is strictly increasing and diverges to infinity.

iii. If $\beta(1-\delta+\theta) < 1$, then the optimal path is strictly decreasing and converges to zero.

Proof. See Appendix B.

Now, we can see that the long-term behavior of the economy depends strongly on patience, represented by β, and the efficiency of human capital formation, represented by θ. If these two values are sufficiently large, sustained growth is ensured.

Obviously, in the intermediate case, where $\beta(1-\delta+\theta) = 1$ for all initial levels of h_0, the size of the economy remains constant.

As an illustrative example, for a logarithmic utility function $u(c) = \ln(c)$ and a Cobb-Douglas function $f(h) = Ah^{\alpha}$ with $0 < \alpha < 1$ and $A > 0$, we can calculate the equation for the optimal path of human capital accumulation[3] as

$$h_t^* = h_0 \beta^t (1 - \delta + \theta)^t \quad for\ all \quad t \geq 0. \tag{28}$$

Sustained growth is equivalent to $\beta(1 - \delta + \theta) > 1$.

4. CONCLUSION

This paper revisits the theoretical framework of endogenous economic growth in response to Lucas (2015), who advocates placing human capital accumulation at the center of economic growth rather than external factors such as technology. We consider the infinite-horizon models in several ways. Our production function of human capital accumulation is generalized throughout the paper to encompass the time and economic effort devoted to investment in education and training and the role of past knowledge in forming the current stock of human capital. Moreover, we show that human capital accumulation can challenge physical capital accumulation as a prime source of economic growth, consistent with the findings of Galor and Moav (2004).

REFERENCES

Amir, R. (1996). Sensitivity analysis of multisector optimal economic dynamics. *Journal of Mathematical Economics*, *25*(1), 123-141. https://doi.org/10.1016/0304-4068(94)00710-1

Caballé, J., & Santos, M. S. (1993). On endogenous growth with physical and human capital. *Journal of Political Economy*, *101*(6), 1042-1067. https://doi.org/10.1086/261914

Galor, O., & Moav, O. (2004). From physical to human capital accumulation: Inequality and the process of development. *The Review of Economic Studies*, *71*(4), 1001-1026. https://doi.org/10.1111/0034-6527.00312

Ha-Huy, T., & Tran, N. T. (2020). A simple characterization for sustained growth. *Journal of Mathematical Economics*, *91*, 141-147. https://doi.org/10.1016/j.jmateco.2020.10.001

Le Van, C., & Dana, R.-A. (2003). *Dynamic programming in economics Vol. 5*. Springer Science & Business Media.

Lucas, R. E., Jr. (1988). On the mechanics of economic development. *Journal of Monetary Economics*, *22*(1), 3-42. https://doi.org/10.1016/0304-3932(88)90168-7

Lucas, R. E., Jr. (2015). Human capital and growth. *American Economic Review*, *105*(5), 85-88. https://doi.org/10.1257/aer.p20151065

[3] In fact, it is a simple task to verify that the sequence $\{h_t^*\}_{t=0}^{\infty}$ satisfies Euler equations and the transversality condition (Amir, 1996). Hence, it is the optimal path.

Manuelli, R. E., & Seshadri, A. (2014). Human capital and the wealth of nations. *American Economic Review*, *104*(9), 2736-2762. https://doi.org/10.1257/aer.104.9.2736

McGrattan, E. R., & Prescott, E. C. (2009). Openness, technology capital, and development. *Journal of Economic Theory*, *144*(6), 2454–2476. https://doi.org/10.1016/j.jet.2008.05.012

Ramsey, F. P. (1928). A mathematical theory of saving. *The Economic Journal*, *38*(152), 543-559. https://doi.org/10.2307/2224098

Schoellman, T. (2012). Education quality and development accounting. *Review of Economic Studies*, *79*(1) 388-417. https://doi.org/10.1093/restud/rdr025

Solow, R. M. (1956). A contribution to the theory of economic growth. *The Quarterly Journal of Economics*, *70*(1) 65-94. https://doi.org/10.2307/1884513

Solow, R. M. (1957). Technical change and the aggregate production function. *The Review of Economics and Statistics*, *39*(3), 312-320. https://doi.org/10.2307/1926047

Stokey, N., Lucas, R. E., Jr., & Prescott, E. C. (1989). *Recursive methods in economic dynamics*. Harvard University Press. https://doi.org/10.2307/j.ctvjnrt76

APPENDIX

Appendix A (Proof of Proposition 2.1)

We want to show that

$$\beta f'(h) - \frac{(1-\beta)\psi'(1)}{h} \geq \frac{\beta}{2} f'(h).$$

For

$$h \geq \bar{h}$$

Equivalently

$$\frac{\beta}{2} f'(h) \geq \frac{(1-\beta)\psi'(1)}{h}.$$

With the assumption $f(h) = h^{\gamma}$, the equation above becomes

$$\frac{\beta}{2}\gamma h^{\gamma-1} \geq \frac{(1-\beta)\psi'(1)}{h},$$

$$\frac{\beta}{2}\gamma h^{\gamma-1} h \geq \frac{(1-\beta)\psi'(1)}{h} h,$$

$$\frac{\beta}{2}\gamma h^{\gamma} \geq (1-\beta)\psi'(1),$$

$$h^{\gamma} \geq \frac{2(1-\beta)\psi'(1)}{\gamma\beta}.$$

Then, we have

$$\bar{h} = \left[\frac{2(1-\beta)\psi'(1)}{\gamma\beta}\right]^{\frac{1}{\gamma}}.$$

Next, we get

$$\int_{\tilde{h}}^{\infty}\big(V_2(h,h)+\beta V_1(h,h)\big)dh = \int_{\tilde{h}}^{\infty} u'\big(f(h)-\psi(1)\big)\left(\beta f'(h) - \frac{(1-\beta)\psi'(1)}{h}\right)dh$$

$$\geq \frac{\beta}{2}\int_{\tilde{h}}^{\infty} u'(f(h)-\psi(1))f'(h)dh$$

$$= \frac{\beta}{2}\lim_{h\to\infty}\big(u(f(h)-\psi(1)) - u(f(\tilde{h})-\psi(1))\big)$$

$$= \infty.$$

Appendix B (Proof of Proposition 3.1)

i) The uniqueness is a direct corollary of the strictly concavity of u and f.

ii) We can consider a variation of problem (P), say (P'):

$$\max\left[\sum_{t=0}^{\infty} \beta^t V(h_t, h_{t+1})\right],$$

$$s.t \quad 0 \leq h_{t+1} \leq (1-\delta+\theta)h_t,$$

$$h_0 \ \ is \ \ given.$$

Now, we will prove that the optimal sequence of problem (P') is feasible for problem (P), hence it is also optimal for problem (P).

Consider the optimal sequence of problem (P'), $\{h_t^*\}_{t=0}^{\infty}$. From the Inada conditions, one has for any t

$$0 < h_{t+1} < (1-\delta+\theta)h_t.$$

First, observe that since $\beta(1-\delta+\theta)>1$, for every h we have $V_2(h,h)+\beta V_1(h,h)>0$. Hence, the steady state does not exist. Consider the Euler equation, which is the same for (P) or (P'):

$$\frac{1}{\theta}u'\left(f\left(\frac{(1-\delta+\theta)h_t^*-h_{t+1}^*}{\theta}\right)\right)f'\left(\frac{(1-\delta+\theta)h_t^*-h_{t+1}^*}{\theta}\right)$$

$$=\frac{\beta(1-\delta+\theta)}{\theta}u'\left(f\left(\frac{(1-\delta+\theta)h_{t+1}^*-h_{t+2}^*}{\theta}\right)\right)f'\left(\frac{(1-\delta+\theta)h_{t+1}^*-h_{t+2}^*}{\theta}\right),$$

which is equivalent to

$$u'\left(f\left(\frac{(1-\delta+\theta)h_t^*-h_{t+1}^*}{\theta}\right)\right)f'\left(\frac{(1-\delta+\theta)h_t^*-h_{t+1}^*}{\theta}\right)$$

$$=\beta(1-\delta+\theta)u'\left(f\left(\frac{(1-\delta+\theta)h_{t+1}^*-h_{t+2}^*}{\theta}\right)\right)f'\left(\frac{(1-\delta+\theta)h_{t+1}^*-h_{t+2}^*}{\theta}\right).$$

Since $\beta(1-\delta+\theta)>1$, this equality implies that the sequence $\{(1-\delta+\theta)h_t^*-h_{t+1}^*\}_{t=0}^{\infty}$ is an increasing function. Since the steady state does not exist, this sequence diverges to infinity. From the supermodularity property, the optimal sequence $\{h_t^*\}_{t=0}^{\infty}$ is monotonic. Moreover, because of $(1-\delta+\theta)h_t^*-h_{t+1}^*$ diverges to infinity, then the sequence $\{h_t^*\}_{t=0}^{\infty}$ is strictly increasing and also diverges to infinity.

iii) For the case $\beta(1-\delta+\theta)<1$, consider the optimal sequence $\{h_t^*\}_{t=0}^{\infty}$ of problem (P'). By the super-modularity property, this sequence is monotonic.

By the Inada condition, for any t, $h_{t+1}^*<(1-\delta+\theta)h_t^*$. If $h_0\le h_1^*$, then $\{h_t^*\}_{t=0}^{\infty}$ is strictly increasing and is an interior solution, which contradicts the Euler equation. Hence the optimal sequence $\{h_t^*\}_{t=0}^{\infty}$ is strictly decreasing. If there exists an infinite number of t such that $h_{t+1}^*=(1-\delta)h_t^*$, then $\lim_{t\to\infty}h_t^*=0$. Suppose that for t sufficiently big, $h_{t+1}^*>(1-\delta)h_t^*$, then by the Euler equation, $(1-\delta+\theta)h_t^*-h_{t+1}^*$ is decreasing. The limit of $(1-\delta+\theta)h_t^*-h_{t+1}^*$ cannot be strictly positive, since this case contradicts the Euler equation. Hence, $\lim_{t\to\infty}(1-\delta+\theta)h_t^*-h_{t+1}^*=0$. This implies $\lim_{t\to\infty}h_t^*=0$.